Chapter 4

Distribution[1]

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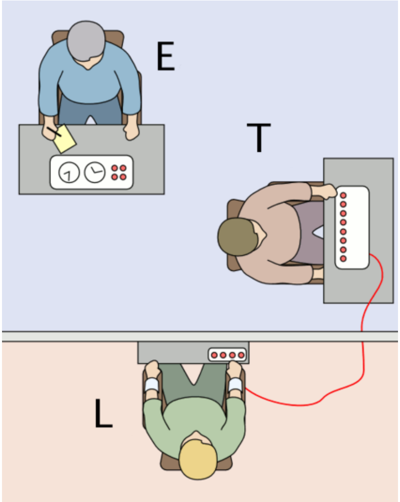
[1] These notes use content from OpenIntro Statistics Slides by Mine Cetinkaya-Rundel.

Geometric distribution

Milgram experiment

- Stanley Milgram, a Yale University psychologist, conducted a series of experiments on obedience to authority starting in 1963.
- Experimenter(E) orders the teacher (T), the subject of the experiment, to give severe electric shocks to a learner (L) each time the learner answers a question incorrectly.
- The learner is actually an actor, and the electric shocks are not real, but a prerecorded sound is played each time the teacher administers an electric shock.



Milgram experiment

- These experiments measured the willingness of study participants to obey an authority figure who instructed them to perform acts that conflicted with their personal conscience.
- Milgram found that about 65% of people would obey authority and give such shocks.
- Over the years, additional research suggested this number is approximately consistent across communities and time.

Bernouilli random variables

- Each person in Milgram's experiment can be thought of as a **trial**.
- A person is labeled a **success** if she refuses to administer a severe shock, and **failure** if she administers such shock.
- Since only 35% of people refused to administer a shock, **probability of success** is $\mathbf{p = 0.35}$.
- When an individual trial has only two possible outcomes, it is called a **Bernoulli random variable**.

Geometric distribution

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... the third person?

$$\text{P}(1^{st} \text{ and } 2^{nd} \text{ shock, } 3^{rd} \text{ refuses})$$
$$= \frac{S}{0.65} \times \frac{S}{0.65} \times \frac{R}{0.35} = 0.65^2 \times 0.35 \approx 0.15$$

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$$\text{P}(1 \text{ shocks, } 10^{th} \text{ refuses})$$
$$= \underbrace{\frac{S}{0.65} \dots \frac{S}{0.65}} \times \frac{R}{0.35} = 0.65^9 \times 0.35 \approx 0.0072$$

Geometric distribution

Geometric distribution describes the waiting time until a success for **independent and identically distributed (iid)** Bernoulli random variables.

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Geometric probabilities

If p represents probability of success, $(1-p)$ represents probability of failure, and n represents number of independent trials

$$\text{P(success on the } n^{th} \text{ trial)} = (1-p)^{n-1}p$$

Practice

Can we calculate the probability of rolling a 6 for the first time on the 6^{th} roll of a die using the geometric distribution? Note that what was a success (rolling a 6) and what was a failure (not rolling a 6) are clearly defined and one or the other must happen for each trial.

A) No, on the the roll of a die there are more than 2 possible outcomes.

B) Yes, why not?

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A) No, on the the roll of a die there are more than 2 possible outcomes.

B) **Yes, why not?**

$$\text{P(6 on the } 6^{th} \text{ roll)} = (\tfrac{5}{6})^5 \cdot (\tfrac{1}{6}) \approx 0.067$$

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But how can she test a non-whole number of people?

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Mean and standard deviation of geometric distribution

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- Dr. Smith is expected to test 2.86 people before finding the first one that refuses to administer the shock, give or take 2.3 people.
- These values only make sense in the context of repeating the experiment many many times.